# AN INTERPRETATION ON PRESSURE ELONGATION IN PIPING SYSTEMS

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ABSTRACT

The elongation of pipe due to internal pressure can have considerable effect on pipe stress analysis. Owing to its significance, the pressure elongation has been included in many popular pipe stress computer programs. However, due to lack of proper interpretation, this so called pressure load case has often been misapplied resulting in unconservative analysis. This paper presents a brief derivation of the pressure elongation, describes the approach adopted by most computer programs, investigates the nature of the stress generated, and explains the method of evaluating the stress in compliance with the Code requirements.

NOMENCLATURE

D - Means diameter of pipe

E - Modulus of elasticity

$e_h$ - Circumferential strain

$e_l$ - Longitudinal strain

P - Internal pressure

R - Radius of curvature of bend

r - Mean radius of pipe = D/2

$S_h$ - Circumferential or hoop stress

$S_l$ - Longitudinal stress

t - Thickness of pipe

$\beta$ - Change of bend angle due to pressure

$\mu$ - Poisson's ratio

$\theta$- Bend angle

## INTRODUCTION

When piping is pressurized, its entire inner surface is subjected to a uniform pressure loading. This pressure loading creates stresses in the directions normal to the wall, parallel to the pipe axis, and tangential to the cross sectional circle. At the same time it elongates the pipe again in the axial and circumferential directions. The stretch in the axial direction is generally referred to as pressure elongation. The effect of pressure elongation in a piping system is equivalent to thermal expansion. It can create high bending stress in systems which do not have adequate flexibility.

The effect of pressure elongation is particularly important in pipeline design. It can surpass the thermal expansion in a cross country transportation pipeline because of the low temperature rise and thin wall construction of the line. Owing to its significance, the pressure elongation has been included in most of the pipe stress computer programs (1,2,3). However, this so called pressure load case has also frequently been mis-interpreted due to the gap between the actual program formulation and the user's conception.

Calculating deflection due to pressure loading in a piping system is a very complicated process. An accurate result can only be obtained by modeling the pipe with three dimensional shell elements. Unfortunately the three dimensional shell element approach is very costly and is not suitable for large volume production analyses. The current pipe stress computer programs are almost entirely based on beam elements. They are not capable of handling internal radial loading and hoop stress. This inability to include the radial pressure load makes the pressure load case less than what would be expected from a three dimensional pressure analysis. Due to this discrepancy, the pressure load case, if misapplied, can result in misleading answers. The pressure elongation is generally insignificant in normal plant piping.

In the following a brief derivation of pressure elongation is made, the approach adopted by most computer programs is described, the nature of the stress generated is investigated and finally the method of evaluating the stress is explained. The article is intended to serve as an interpretation; therefore, effort is placed mainly on presenting the physical pictures rather than the rigorous mathematical derivations.

## PRESSURE ELONGATION

In a pressurized pipe, the entire inside wall sustains a uniform pressure P as shown in Figure 1. This pressure loading develops a tri-axial stress in the pipe wall. The stress component normal to the pipe wall is generally small and will be ignored in this discussion. The pipe is considered to be subject to stresses $S_h$ and $S_1$ in circumferential and longitudinal directions respectively. From Hook's law and Poisson's ratio the net strains in these two directions can be written as (4):

$$e_h = \frac{S_h}{E} - \frac{\mu S_1}{E} \tag{1}$$

$$e_1 = \frac{S_1}{E} - \frac{\mu S_h}{E} \tag{2}$$

The first term of the right hand side of Equations (1) and (2) is the elongation due to the direct stress in a given direction, while the second term is the shrinkage in a given direction caused by the elongation in the perpendicular direction. The elongation in one direction depends on the stress in that direction as well as the stress in the perpendicular direction.

### (a) Straight Pipe

The pressure stress in a thin wall straight pipe can be determined by the

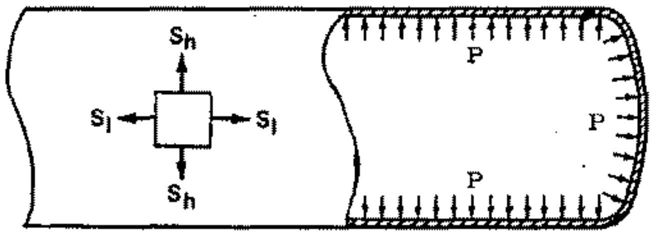

Figure 1. Pressure Loading

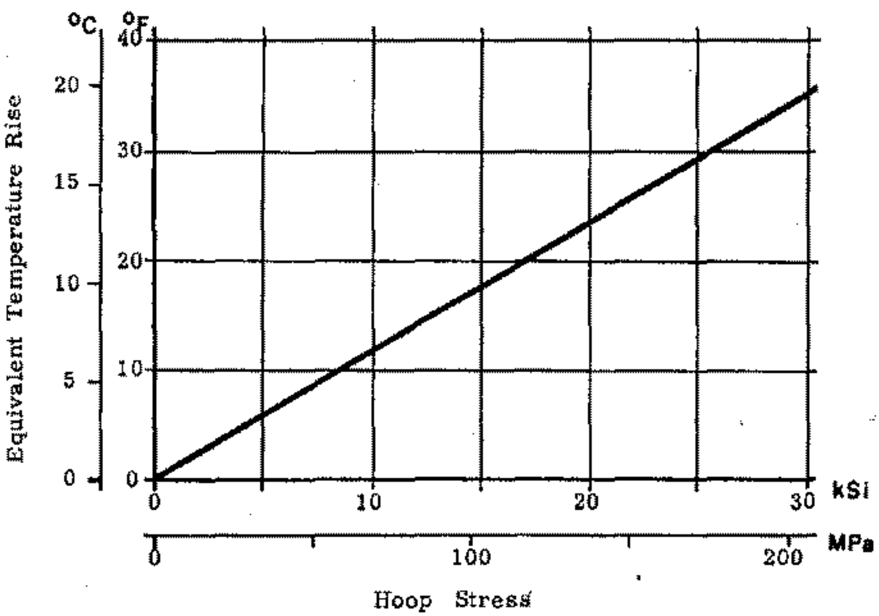

Figure 2. Significance of Pressure Elongation

static equilibrium between the pressure force and the stress force acting on a symmetrically cut-off section. That is (4):

$$S_h = PD/2t \quad (3)$$

$$S_1 = PD/4t \quad (4)$$

Substituting Equations (3) and (4) in Equation (2), the longitudinal pressure elongation strain becomes,

$$e_1 = \frac{PD}{4tE}(1 - 2\mu) \quad (5)$$

The circumferential elongation, which is irrelevant to this discussion, is omitted.

The significance of the pressure elongation can be better visualized by converting it to equivalent temperature. Figure 2 shows the relation between pressure elongation in term of hoop stress, and equivalent temperature rise for low carbon steel pipe. For plant piping the hoop stress is normally maintained at less than 15000 psi (103.4MPa) which is equivalent to a temperature increase of 17.5°F (9.72°C). In view of the high temperature range normally experienced in plant piping, the effect of pressure elongation in plant piping is insignificant. On the other hand, the hoop stress in a cross country transportation pipeline can reach 30000 psi (206.8MPa) or higher for high strength pipe. The equivalent temperature rise due to pressure elongation can easily exceed 35°F (19.44°C) which represents a high percentage of the design temperature rise. Therefore, the pressure elongation is an important factor to be considered in transportation pipeline design.

(b) Bend

The pressurization at an elbow or bend is a complicated process. Even for thin wall bends there is no fully satisfactory stress analysis for the effects of internal pressure (5). As shown in Figure 3, when a segment of bend K-L-M-N is pressurized it moves to K'-L'-M'-N'. Its dimensions, bend angle, and bend radius all change. To investigate the bend deformation, the first step is to establish the stresses in both circumferential and longitudinal directions. Take the section a-b-c-d for instance, intuitively we know the longitudinal stress is the same as in the case of straight pipe. If an imaginary section is cut along b-c we can hardly tell the difference between the section and a straight pipe section. However, the hoop stress is somewhat different. By cutting off along the plane of symmetry, the pipe wall a-b and c-d will have to absorb the pressure acting at the shaded area. Comparing with an equivalent straight pipe section drawn with parallel dotted lines, it is easily pictured that a-b takes more stress than in the case of straight pipe while c-d takes less. By considering the bend as a continuous torus, it can be shown that the membrane stresses (average through the thickness of pipe) due to internal pressure are (6,7):

$$S_h = \frac{PD}{2t} \cdot \frac{2R - r}{2(R - r)} \quad \text{at } \widehat{K' - L'} \quad (6a)$$

$$S_h = \frac{PD}{2t} \cdot \frac{2R + r}{2(R + r)} \quad \text{at } \widehat{M' - N'} \quad (6b)$$

$$S_1 = PD/4t \quad (7)$$

The next step is to construct a physical picture to see how the bend deforms. When pressure is introduced to the bend, it stretches the pipe wall in radial as well as in longitudinal directions. If the elongation rate is the same in all directions, the case becomes a rigid body expansion just like in thermal expansion. The bend angles remains the same ($\theta' = \theta$) but the bend radius increases ($R' > R$). In the real case, the elongation rate in the radial direction is greater than that in the longitudinal direction due to higher mem-

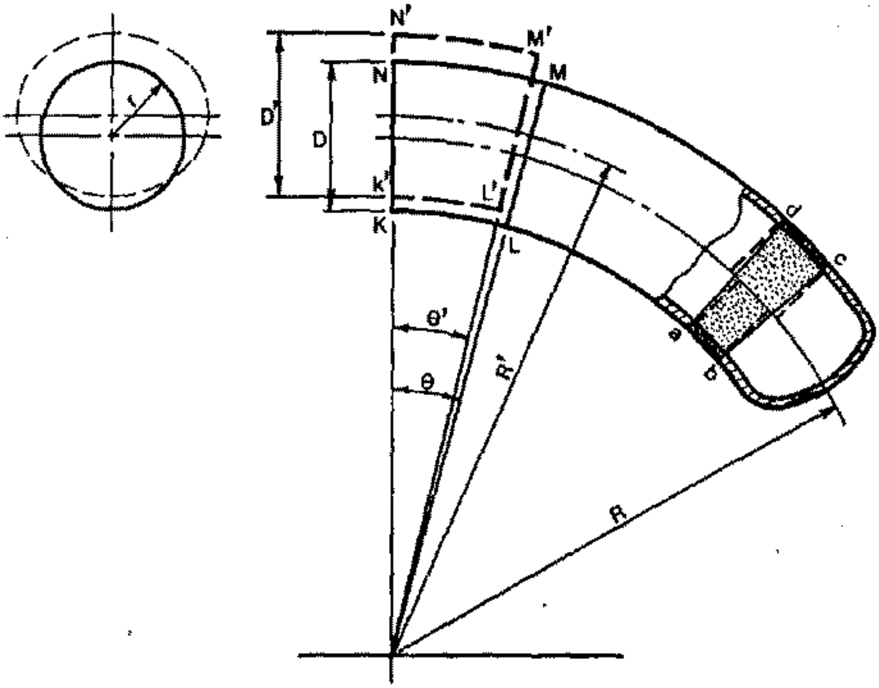


Figure 3. Pressurization of Bends

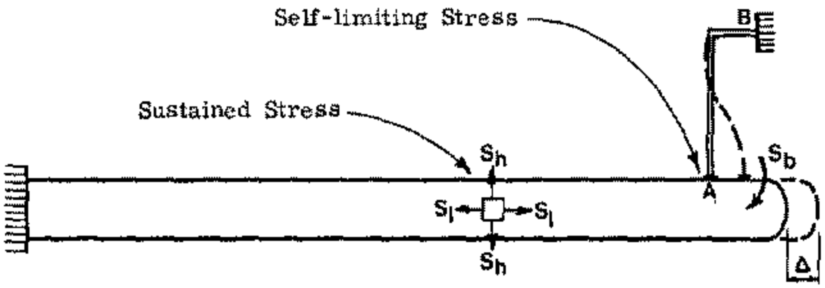


Figure 4. Stresses Caused by Internal Pressure

brane stress in the circumferential direction. The bend will tend to flatten making the bend angle somewhat smaller and the bend radius proportionally larger.

The change in bend angle and the change in bend radius can be estimated by assuming the average hoop stress $S_h = PD/2t$, we have

$$D' \doteq D\left(1 + \frac{PD}{2tE} - \frac{\mu PD}{4tE}\right) \quad (8)$$

$$\widehat{K'-L'} = (R-r)\theta\left(1 + \frac{PD}{4tE} - \frac{\mu PD}{2tE} \cdot \frac{2R - r}{2(R - r)}\right) \quad (9)$$

$$\widehat{M'-N'} = (R+r)\theta\left(1 + \frac{PD}{4tE} - \frac{\mu PD}{2tE} \cdot \frac{2R + r}{2(R + r)}\right) \quad (10)$$

The new bend angle is then determined by

$$\theta' = \frac{\widehat{M'-N'} - \widehat{K'-L'}}{D'} = \frac{1 + (1 - \mu)PD/4tE}{1 + (2 - \mu)PD/4tE}\,\theta \quad (11)$$

The change in bend angle is

$$\beta = \theta' - \theta = -\frac{PD}{4tE + (2 - \mu)PD}\,\theta \quad (12)$$

The bend angle decreases meaning the pressure tends to open up the bend. However, this opening up effect should not be over emphasized. For example, a forged elbow may have a thicker wall at the crotch area which would greatly reduce the opening effect. The cross section of the bend will also ovalize (7) as shown in Figure 3 which further resists opening. In fact tests have shown (8) that a miter bend when pressurized will close (shutting) the bend angle in the elastic region. Owing to the above facts and uncertainties the application of bend opening effect varies with computer programs. Some programs (1) ignore the opening effect and consider only the uniform elongation while others (2,3) consider varying degrees of the opening effect.

ANALYTICAL APPROACH

A complete pressurization analysis requires the application of the uniform pressure load normal to the entire internal surface. The whole system has to be modeled with numerous small three dimensional shell elements. This method, may be feasible in designing critical vessels, but is very impractical for performing piping stress analysis. Moreover, a satisfactory element yet has to be developed for simulating the bend distortion.

In piping stress analysis, the systems are considered as a series of beams which are not capable of handling the internal radial pressure. The pressure load is seldom applied directly to the system to achieve the elongation effect. Even the longitudinal pressure, which is within the beam element capability, can not be applied directly otherwise the system will give incorrect elongation of PD/4tE instead of the elongation given by Equation (5).

The analytical approach adopted by most of the computer programs ignores completely the origin of the pressure and considers only the net distortion caused by the pressure. The programs work with Equations (5) and (12) and consider these elongations the same way as they consider thermal expansion. The so called pressure load case is somewhat misleading and should be more accurately termed as pressure elongation load case.

NATURE OF STRESS

When piping is pressurized, the pipe shell develops stresses to resist the pressure. The stresses, $S_h$ and $S_l$ in circumferential and longitudinal directions respectively, required for static equilibrium with the pressure are the sustained stresses. Their magnitudes remain unchanged even if the pipe yields. An overstress will cause the yield to continue until either a new equilibrium is reached or the pipe fails.

In addition to developing the sustained stresses in the shell, the pressure also creates elongation. Take Figure 4 as an example; the elongation of the main line probably has very little effect on the main line itself but will have significant effect on the branch line. The stress in the branch created by the pressure elongation of the main line is self-limiting. Once the amount of elongation, $\Delta$, is reached the pipe will stop growing. In the branch line, although high stress may be developed, the yielding, if any, will stop as soon as the displacement at point A reaches the fixed elongation value. Therefore, it is clear that the stress resulting from pressure elongation is not necessarily a sustained stress.

The sustained stress can be calculated with simple Equations (3) and (4). Stress intensification factors may need to be included to compensate for the uneven stress distribution at some special components. They are usually straight forward. On the contrary, the self-limiting stress caused by the pressure elongation which is analogous to thermal expansion is very difficult to calculate. A computerized analysis is generally used to check this pressure elongation effect. The so-called pressure load case is created for this purpose.

CODE COMPLIANCE

The purpose of stress analysis is to ensure that the piping system is safe and complies with Code requirements. It is necessary to find out how the Code requirements can be satisfied with the pressure load case that actually considers only the pressure elongation effect.

The Piping Codes (9, 10) have set different criteria for evaluating sustained and self-limiting stresses because of different factors considered in the analysis. To satisfy the maximum allowable level for the sustained stress one has to calculate the stresses caused by pressure, weight, and other sustained loads. This often tempts an analyst to use a single weight plus pressure load case and to compare the stress printout directly against the Code allowable stress. This approach can lead to nonconservative analysis due to the following:

(a). The weight plus pressure load case is actually the weight plus pressure elongation load case. The stress printed out in some of the programs comes directly from the member forces and moments. The sustained longitudinal pressure stress is not included.

(b) The member forces and moments created by weight loading may cancel with those produced from pressure elongation at some of the points. This creates some inconsistency in the analysis though it is not serious.

One of the acceptable methods is to run a weight load case and a thermal plus pressure elongation load case. The longitudinal weight stress is then either manually or automatically added to the longitudinal sustained pressure stress which is calculated explicitly using Equation (4). The combined stress is used to compare with the Code allowable sustained stress. In the evaluation of self-limiting stresses, thermal expansion and pressure elongation can be combined in one single load case, this is acceptable due to the similarity between these two displacement loadings. However, it should be noted that in designing low temperature lines the inclusion of pressure elongation will result in nonconservative analysis. The low temperature but unpressurized condition gives higher expansion (or contraction) stress and should be used.

CONCLUSIONS

The so called pressure load case analysis as given by many computers programs is in fact more accurately a pressure elongation load case. The result given by this load case is generally less than what is expected from a full pressure load case analysis. This load case can be very easily misapplied resulting in nonconservative analysis. Therefore it is important to know exactly what the computer program is doing.

Pressure elongation has a significant effect on long pipeline design but has only a minor effect on process plant piping. The approach adopted by most computer programs in handling pressure load is to decouple the sustained pressure effect from the elongation effect. The sustained stress can be easily calculated by the simple formula while the elongation effect is normally analyzed by computer methods in a way similar to thermal flexibility analysis. The sustained pressure stress is then added to stresses created by other sustained loads such as weight and seismic forces. The stress generated by pressure elongation is combined with thermal expansion and other displacement stresses. This decoupled analysis is entirely satisfactory. In fact if one is to comply with the Code requirements, in separating sustained and displacement stresses, the decoupled analysis is probably the only approach that should be used.

The inclusion of pressure elongation does not necessarily result in a more conservative analysis. In low temperature piping such as LNG lines, the inclusion of pressure elongation will off-set the piping contraction and give lower total displacement stress. Since the piping can reach or stay at a low temperature condition when pressure is relieved, the analysis should be done without the pressure elongation effect.

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